# Magnetic Force Analysis in a Gapped-Core Reactor Model under Harmonic Magnetizations by Efficient Frequency-Domain Decomposition

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The magnetic force analysis in a gapped-core reactor model under harmonic magnetizations is carried out using efficient decomposition in discrete frequency domain. The nonlinear system of equations is decomposed so that the unknown potentials in the frequency domain are solved separately and in parallel, which benefits to solve large and multi-scale engineering field problems. Harmonic analysis of flux distribution and magnetic force is achieved through numerical computation of the nonlinear magnetic field. The variation of the magnetic force with the size of the air gap is analyzed in order to investigate the influence of the leakage flux on the magnetic force.

Index Terms—Discrete frequency domain, gapped core, harmonic decomposition, magnetic force.

### I. INTRODUCTION

THE ultra high voltage direct current (UHVDC) transmission has been planned and developed in China in recent years. As key electrical devices, large reactors and converter transformers play an important role in the operation of UHVDC [1]. Meanwhile, the existence of rich harmonics in the exciting current or driving voltage leads to distorted flux densities in the saturated iron core. Consequently the magnetic force due to the leakage flux not only exhibits special spectrum in the frequency domain but also threatens the normal operation of electrical devices e.g. vibration and deformation of windings [2-3]. Therefore it is important to investigate the influence of magnetic force on iron core and windings when electrical devices such as reactors work under harmonic magnetizations.

The time-stepping method starting from arbitrary initial value is usually used for the computation of time-periodic nonlinear problems. However, the main drawback is that many periods are required to be stepped through for steady state solution, especially when a much larger time constant exists in the nonlinear system to be analyzed. Numerical methods in time domain and frequency domain have been presented and developed to solve the steady state nonlinear magnetic field efficiently [4-5] by combining the fixed point technique with the finite element method. Different strategies have been presented and investigated to guarantee and even speed up the convergence of solutions [6-7].

In this paper, the harmonic-balance system of equations is established first by approximating all time-periodic variables by complex series. Thereupon, decomposition of the system of equations is achieved according to diagonal dominant characteristic of the reluctivity matrix so that harmonic solutions can be decoupled and computed separately and in parallel. This method is used to compute the magnetic force in a gapped-core model under harmonic magnetizations based on the harmonic solutions of magnetic field. The validation results show the applicability of the proposed method, which is beneficial to solve large-scale engineering field problems.

## II. GAPPED IRON CORE FOR EXPERIMENT

A gapped-core reactor model made of grain-oriented silicon steel sheet (B30P105) produced by Baosteel is used in experiment for comparison of magnetic forces when the model is magnetized by sinusoidal and harmonic excitation. Fig.1 shows the gapped laminated core and the experimental setup.

The thickness of the laminated core is 60 millimetres (mm) and the width of the air gap in the middle limb is 1.8 mm. There are two exciting coils in series on the side yoke. The number of turns of each coil is 115.

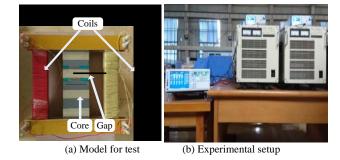


Fig. 1 The gapped core reactor model (a) and experimental setup (b)

#### III. DECOMPOSITION SCHEME IN FREQUENCY-DOMAIN

The two-dimensional nonlinear magnetic field can be formulated as follows by using the magnetic vector potential A,

$$\nabla \times v \nabla \times \boldsymbol{A} + \sigma \frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{J}$$
(1)

where J is the impressed current density,  $\sigma$  and v are the conductivity and reluctivity, respectively.

Owing to the time periodicity of the electromagnetic field under harmonic excitations, periodic variables such as current density J, magnetic vector potential A and reluctivity v can be represented by a summation of trigonometric function,

$$W(t) = \sum_{n=-N_h}^{N_h} W_n e^{jn\omega t}$$
(2)

$$\boldsymbol{W}_{f} = \begin{bmatrix} W_{0} & W_{-1} & W_{1} & \cdots & W_{-N_{h}} & W_{N_{h}} \end{bmatrix}$$
(3)

where W can be replaced by J, A and v.  $N_{\rm h}$  is the total number of harmonics truncated in computation and  $\omega$  is the angular frequency.  $W_n$  is the *n*-th component in frequency domain.  $W_f$ is the spectrum of the periodic variable W.

Actually the time-domain multiplication of magnetic vector potential A and reluctivity v in (1) is equivalent to the frequency-domain convolution of the two variables, therefore a new system of equations in the frequency domain can be obtained, as follows, by using the harmonic-balance theory [8] as well as applying Galerkin's method and finite element method over the whole problem domain,

$$\boldsymbol{M}(\boldsymbol{\sigma},\boldsymbol{H})\boldsymbol{A}_{f} + \boldsymbol{S}(\boldsymbol{D})\boldsymbol{A}_{f} = \boldsymbol{K}_{f}$$
(4)

where M is the mass matrix and S the stiffness matrix. H and D are, respectively, the harmonic matrix and the reluctivity matrix [9].  $K_f$  is related to spatial distribution of the impressed current density.  $A_f$  is the vector potential in frequency domain.

The convolution product term including  $v_0$ , which is dc component of reluctivity, is often dominant, therefore the other terms can be assumed as locally constant and moved to the right-hand side. Consequently, a new decomposed harmonic-balance system of equations can be obtained to decouple harmonic solutions,

$$\left[jn\omega\boldsymbol{M}(\sigma) + \boldsymbol{S}(v_0)\right]\boldsymbol{A}_{f,n} = \boldsymbol{F}_{f,n} + \boldsymbol{K}_{f,n}\left(n = -N_h...N_h\right)$$
(5)

$$\boldsymbol{K}_{f,n} = \iint_{\Omega} \left( \boldsymbol{J}_{f,n} \cdot \boldsymbol{N}_i \right) dS \tag{6}$$

$$\boldsymbol{F}_{f,n} = \sum_{m=-N_h}^{N_h} \boldsymbol{S}(\boldsymbol{v}_{n-m}) \boldsymbol{A}_{f,m}$$
(7)

where  $A_{f,m}$  and  $J_{f,n}$  are the *m*-th harmonic solution of magnetic vector potential and *n*-th harmonic vector of impressed current density.  $N_i$  is the shape function on the *i*-th node in the finite element region  $\Omega$ .  $F_{f,n}$  is obtained from the convolution product of reluctivity and vector potential in harmonic domain.

The decomposed system of equations can be solved separately and in parallel. Meanwhile only  $N_{\rm h}+1$  equations in (5) are required to be solved due to the conjugate symmetry of the harmonic solutions.

#### IV. COMPUTATION AND RESULTS

The gapped-core reactor model in Fig. 1 is used for the computation of the nonlinear magnetic field and the corresponding magnetic nodal force [10]. A sinusoidal exciting current of 50 Hz is applied to make the model operate in nonlinear region. Nonlinear iteration is stopped when the mean and the maximum relative variations of reluctivity become smaller than 0.0001 and 0.01, respectively. Variations of reluctivity in nonlinear iteration are shown in Fig. 2. One node on the interface between the iron core and the air gap is selected to compare the calculated magnetic nodal forces  $(F_{y})$ by the proposed method in this paper and the time-stepping method in MagNet, Infolytica, which is shown in Fig. 3.

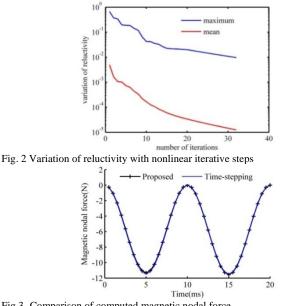


Fig.3. Comparison of computed magnetic nodal force

# V.CONCLUSION

An efficient frequency-domain decomposition method is presented to analyze the magnetic force resulted from harmonic magnetizations. Comparison of computed magnetic nodal forces by the proposed method and the time-stepping method shows good consistency, which proves the accuracy of the decomposition method in the frequency domain. More results under harmonic excitations and the relevant detailed analysis and validation will be presented in the extended paper.

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